Network Interference Management: Understanding Signal Dimensions

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Wireless in Yesterday and Today

$1G$ Analog Systems

$2G$ Digital Systems

$2.5G$ Evolution to 3G

$3G$ IMT-2000

$B3G/4G$ IMT-Advanced

⋮

Higher Data Rate

1948 Shannon Theory

(single-user point-to-point channel)

⋮

1980 Network Info. Theory (Period I)

1990

LDPC / Turbo

2000

2010 Network Info. Theory (Period II)

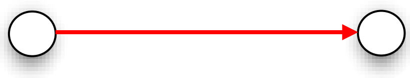
⋮

Explore the capacity limits of communications

Practice ⟵ gap ⟶ **Theory**

Capacity and Degrees of Freedom

Point to Point AWGN Channel



$$\begin{aligned} C &= \log(1+\mathrm{SNR}) \quad \text{bits per signaling dimension} \\ &\approx \log(\mathrm{SNR}) \end{aligned}$$

Bandlimited Channel

- Sampling Theorem

Any signal limited to bandwidth “B” can be expressed by “B” freely chosen complex samples per second.

$$\begin{aligned} C &= B \log(1+\mathrm{SNR}) \quad \text{bits per sec} \\ &\approx B \log(\mathrm{SNR}) \end{aligned}$$

pre-log = bandwidth = degrees of freedom = signaling dimensions

number of independent data streams

Signal Dimensions (Degrees of Freedom):

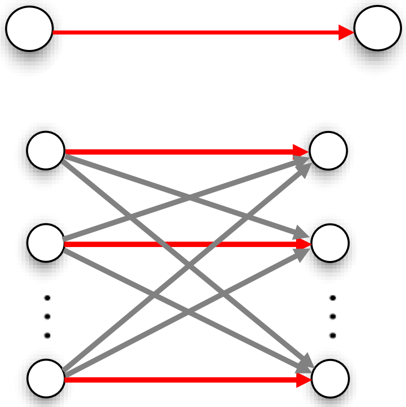
First order approximation to the capacity of Gaussian wireless networks.

From Single-user to Multiuser

Obstacles

- Noise

- Interference

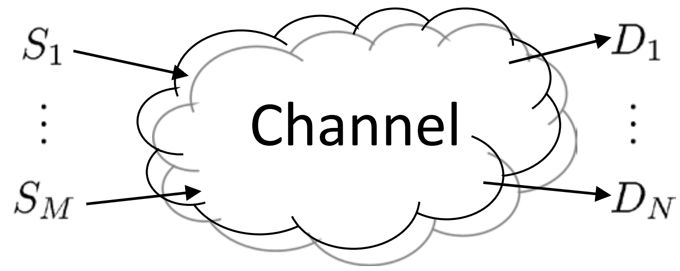


- Competition among users for resources creates interference

- Interference creates a bottleneck for communication rates

- What is the **optimal** strategy to deal with interference?

Why is Signal Dimension (DoF) Analysis Important?



We are more interested in the signaling dimension (DoF) analysis of networks.

(1) Not the end itself, but the means to an end.

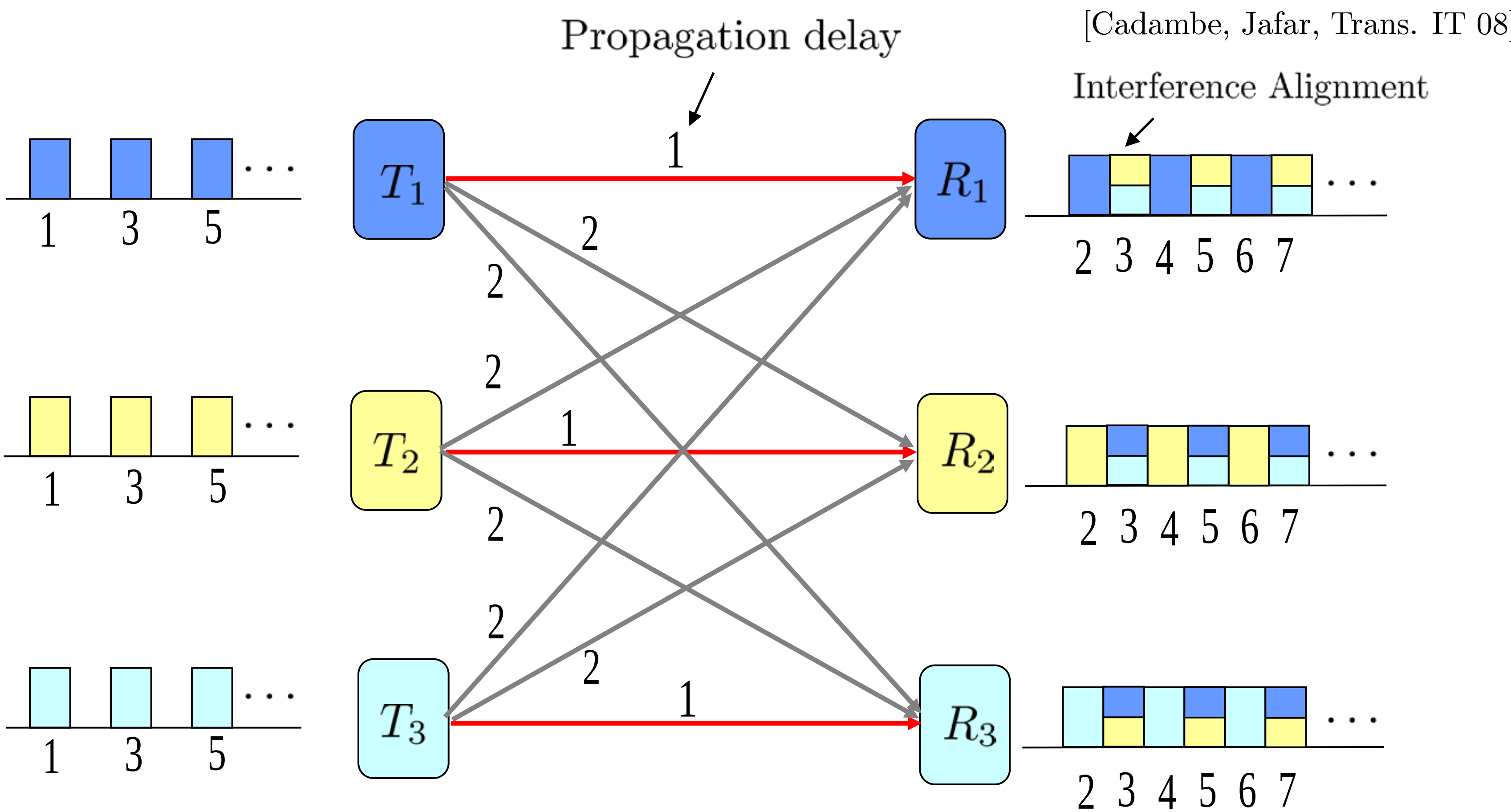
(2) Indicate where the largest gap exists in our understanding the nature of the fundamental capacity limits.

(3) Where the biggest impact ideas may emerge out.

(4) For linear networks (noiseless), it leads to exact capacity.

(finite field channel, index coding, network coding)

Interference Alignment - Toy Example



Each speaker can only speak for one third of the time?

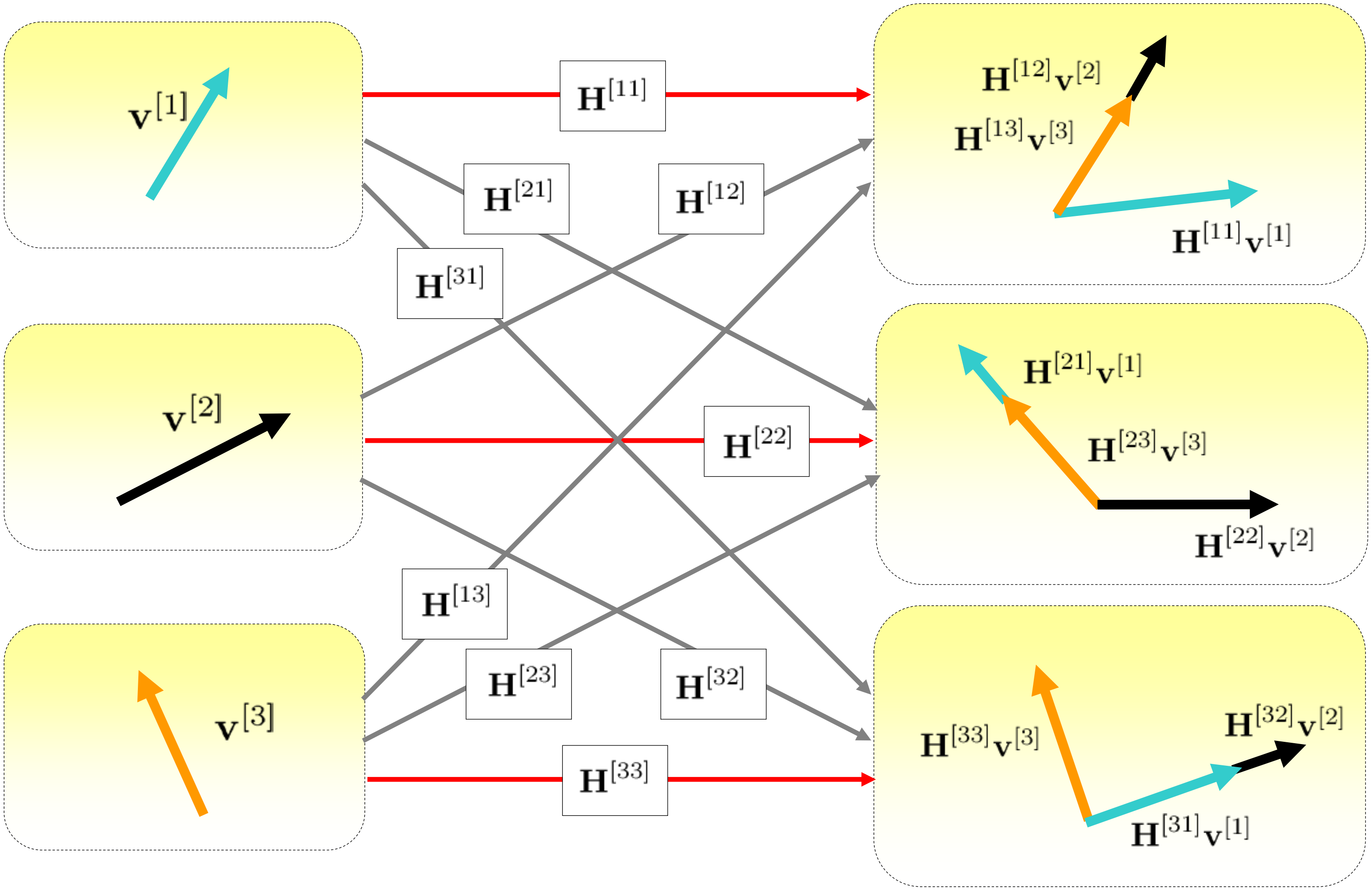
Each speak can speak for half of the time!

Everyone gets half of the cake! **(even for more than 3 users)**

CSIT is required !

3-User MIMO IC (2 antennas at each node)

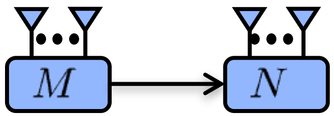
[Cadamber, Jafar, 2008]



Without interference alignment, each user achieves DoF = 2/3.

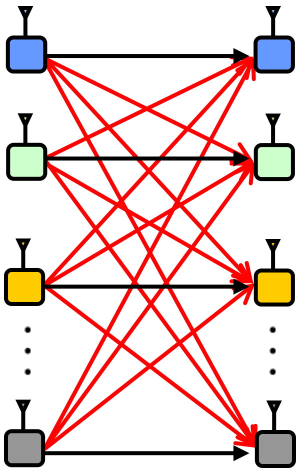
With interference alignment, each user achieves DoF = 1, half of its interference-free DoF.

Explore the Signaling Dimensions



MIMO/P2P, BC, MAC

$\text{DoF} = \min(M, N)$



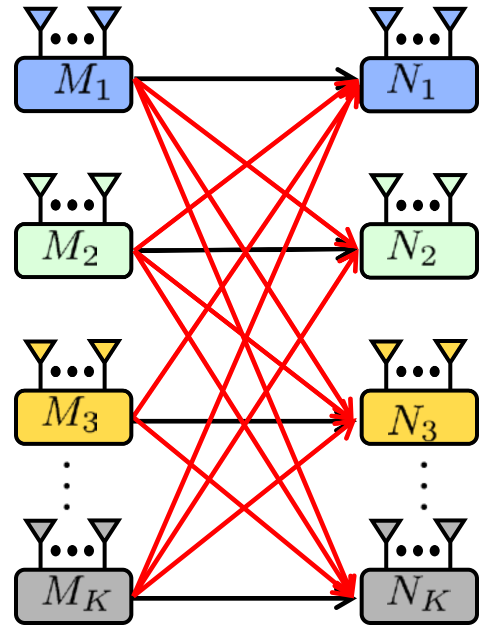
SISO

IC: $\text{DoF} = \frac{K}{2}$

[Cadambe, Jafar, 2008]

X: $\text{DoF} = \frac{K^2}{2K-1}$

[Cadambe, Jafar, 2008]



MIMO IC

DoF = ?

Open in general!

We are interested in signaling dimension analysis of a network:

(1) If the channel coefficients are constant and complex-valued in K User SISO interference channel?

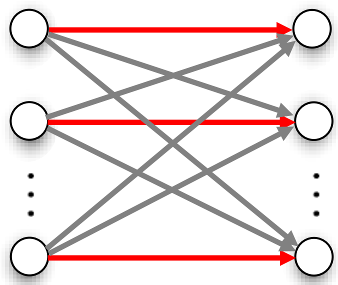
(2) What is the DoF value of the K User MIMO interference channel?

(3) If there is channel uncertainty?

Host-Madsen-Nosratinia Conjecture

[Høst-Madsen, Nosratinia, ISIT 05]

Conjecture: Consider Gaussian interference channels with 3 or more users and constant and complex-valued channel coefficients. These channels have only one DoF regardless of the number of users, almost surely.



$$\cancel{\mathrm{DoF} = 1}$$

Except for special cases, this problem is open in general!

Contribution: Settling this conjecture in the negative.

Approach: Beamforming solution.

Combine symbol extension and asymmetric complex signaling.

What is Asymmetric Complex Signaling?

Consider an $M \times 1$ Gaussian random vector $\mathbf{X}$

Covariance matrix: $\mathbf{K} = \mathbb{E}[\mathbf{X}\mathbf{X}^\dagger]$

Pseudo-Covariance matrix: $\mathbf{K}' = \mathbb{E}[\mathbf{X}\mathbf{X}^T]$

Proper Gaussian: $\mathbf{K}' = \mathbf{0}$

Improper Gaussian: $\mathbf{K}' \neq \mathbf{0}$

$$\mathbf{x}^{[k]} = \begin{bmatrix} \Re\{x_1^{[k]}\} + j\Im\{x_1^{[k]}\} \\ \Re\{x_2^{[k]}\} + j\Im\{x_2^{[k]}\} \\ \vdots \end{bmatrix}$$

$$\mathbf{Q}_k = \mathbb{E}[\mathbf{x}^{[k]}\mathbf{x}^{[k]\dagger}] \qquad \mathbf{X} \sim \mathcal{CN}(\mathbf{0}, \mathbf{K}).$$

Intuitively, the real part and imaginary part have independent identically distribution.

$$\overline{\mathbf{x}}^{[k]} = \begin{bmatrix} \Re\{x_1^{[k]}\} \\ \Im\{x_1^{[k]}\} \\ \Re\{x_2^{[k]}\} \\ \Im\{x_2^{[k]}\} \\ \vdots \end{bmatrix}$$

$$\overline{\mathbf{Q}}_k = \mathbb{E}[\overline{\mathbf{x}}^{[k]}\overline{\mathbf{x}}^{[k]\dagger}]$$

depends on both $\mathbf{K}$ and $\mathbf{K}'$

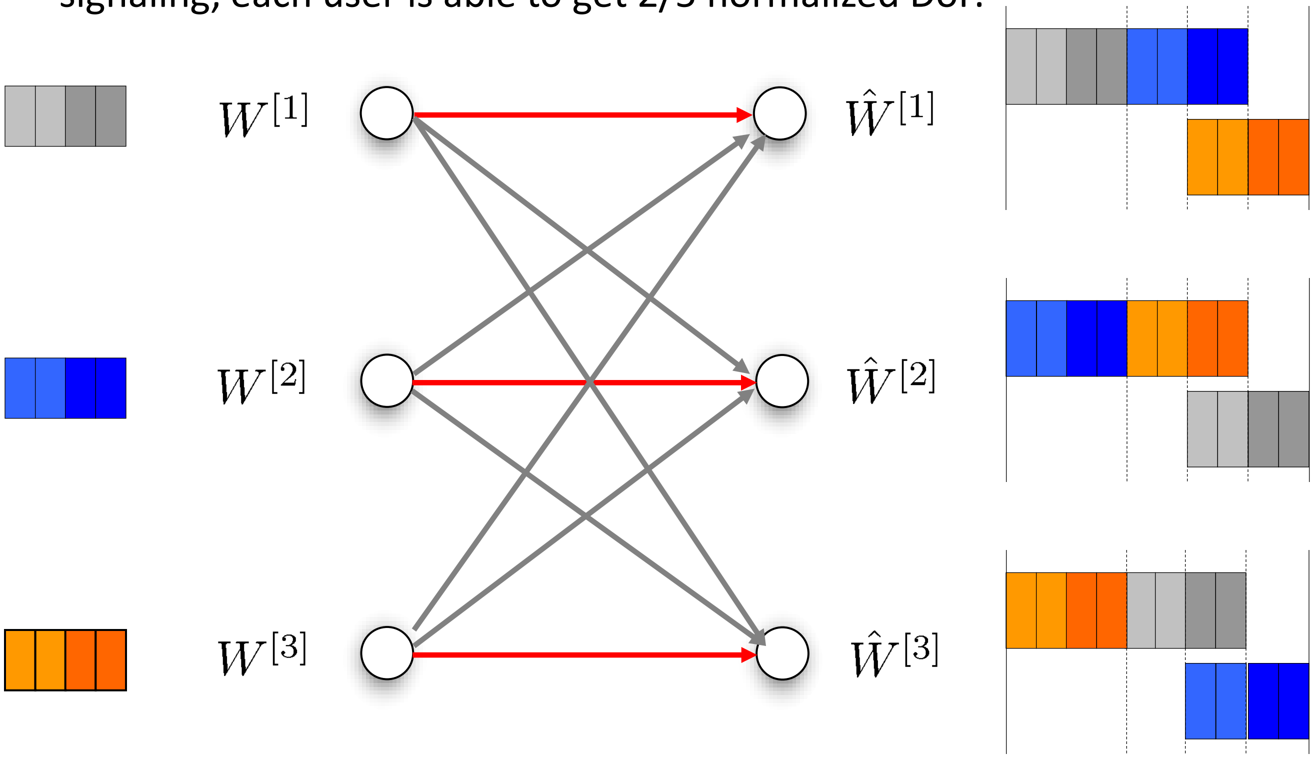
$$y = hx + z \quad \Rightarrow \quad \begin{bmatrix} \Re\{y\} \\ \Im\{y\} \end{bmatrix} = \underbrace{\begin{bmatrix} \Re\{h\} & -\Im\{h\} \\ \Im\{y\} & \Re\{h\} \end{bmatrix}}_{\text{rotation matrix}} \begin{bmatrix} \Re\{x\} \\ \Im\{x\} \end{bmatrix} + \begin{bmatrix} \Re\{z\} \\ \Im\{z\} \end{bmatrix}$$

Setting the Conjecture in the Negative

[Cadamber, Jafar, Wang, 2009]

New result: DoF$= 1.2$

Achievability: With 5 symbol extensions and interference alignment with asymmetric complex signaling, each user is able to get 2/5 normalized DoF.

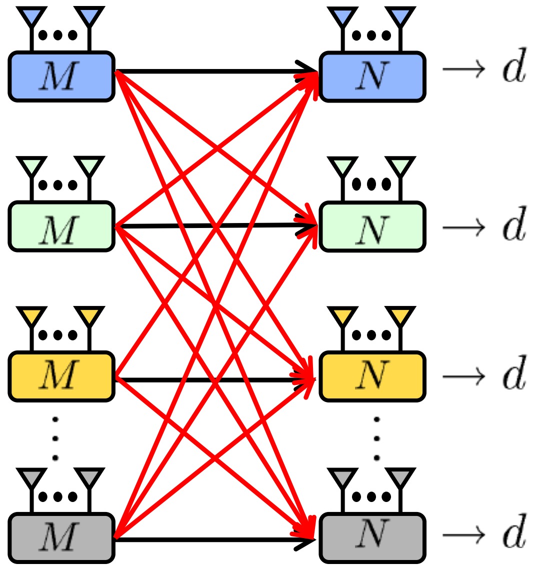


Converse : The signal aligned at one unintended receiver cannot be aligned at the other unintended receiver.

$$\text{DoF} = 4 \times 3 \times \frac{1}{2} \times \frac{1}{5} = \frac{6}{5}$$

New Insight: Phase Alignment

K User $M \times N$ MIMO IC: $(M, N, d)^K$



M antennas at each transmitter

N antennas at each receiver

Full channel knowledge

Information theoretic DoF?

Classification:

Under constrained (Structure independent linear schemes)

Channel is a generic linear transformation between the input space and the output space, lacking any special structure.

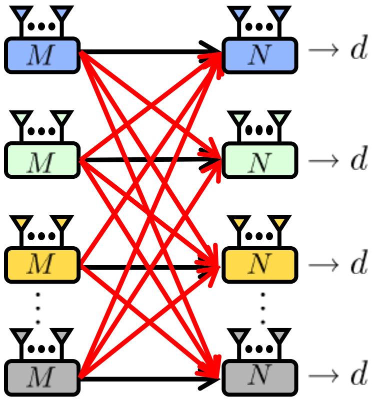
(one channel use, symmetric signaling)

Over constrained (Structure dependent linear schemes)

Channel has structure (multiple channel uses, time/freq varying)

(asymptotic interference alignment)

K User $M \times N$ MIMO IC: $(M, N, d)^K$



Full channel knowledge
Fully connected
Generic channels

Unstructured Linear Schemes

Counting Bound $d \leq \frac{M+N}{K+1}$

[Yetis et.al.]
[Bresler et.al.]
[Razaviyayn et.al.]

(DoF per user scales as $\frac{1}{K}$)

structure independent linear schemes are interference limited

Is this outer bound achievable?

Structured Linear Schemes

Decomposition Bound $d \geq \frac{MN}{M+N}$

[Gou, Jafar, et. al.]
[Ghasemi et.al.]

[CJ08] Asymptotic Interference Alignment

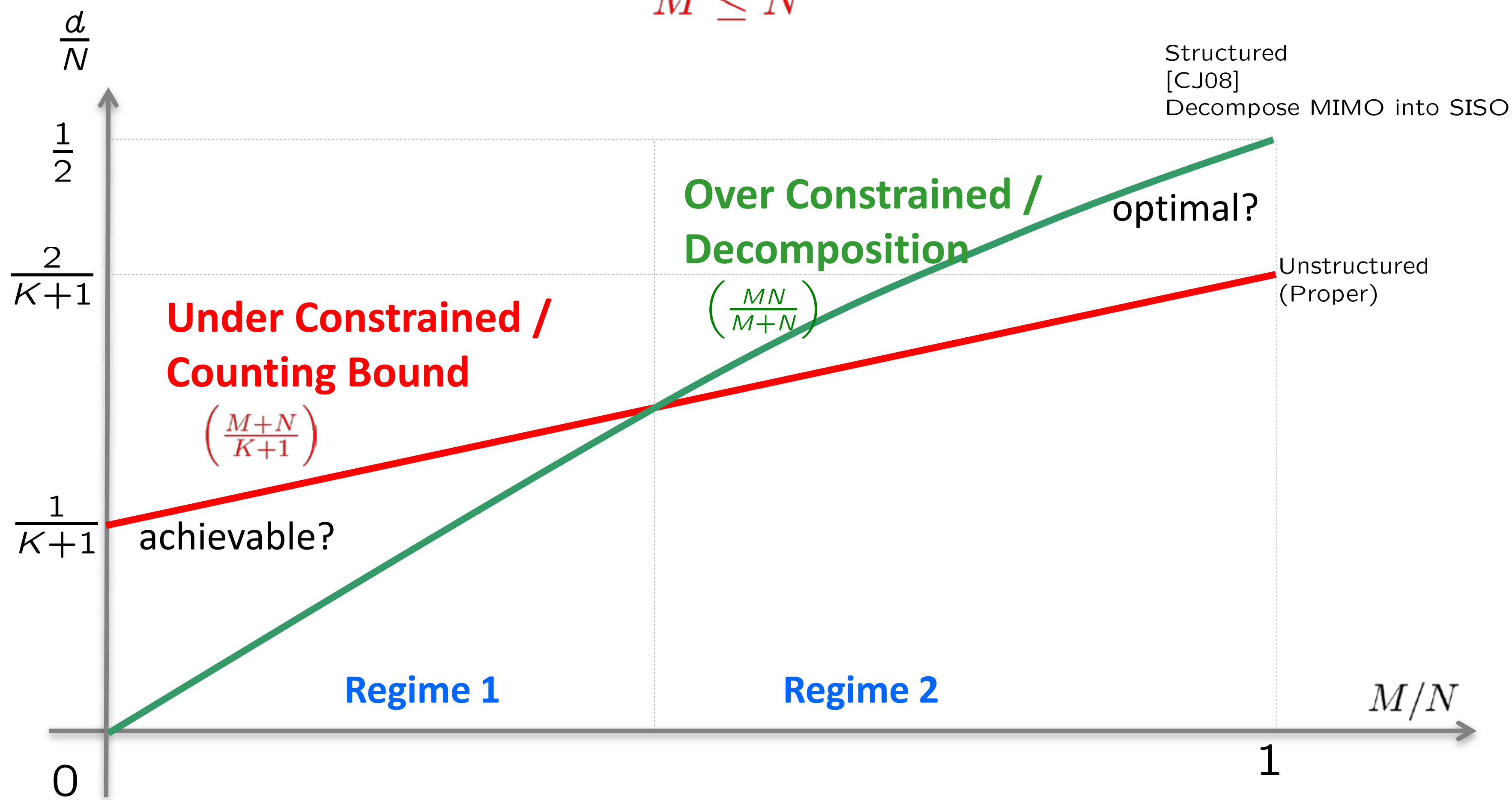
Not interference limited.

Decomposes multiple antenna nodes into multiple single antenna nodes

Is this inner bound optimal?

K User $M \times N$ MIMO IC

$M \leq N$



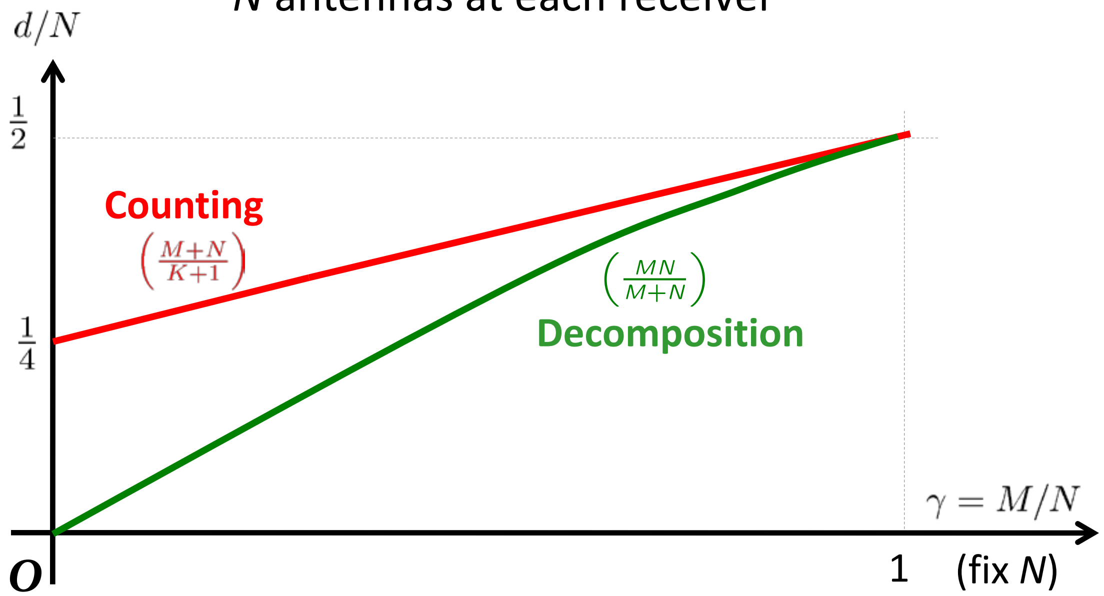
Understanding Regime 1 and Regime 2: Information Theoretic DoF?

Understanding Regime 1: Information Theoretic DoF

K = 3 User MIMO Interference Channel

M antennas at each transmitter

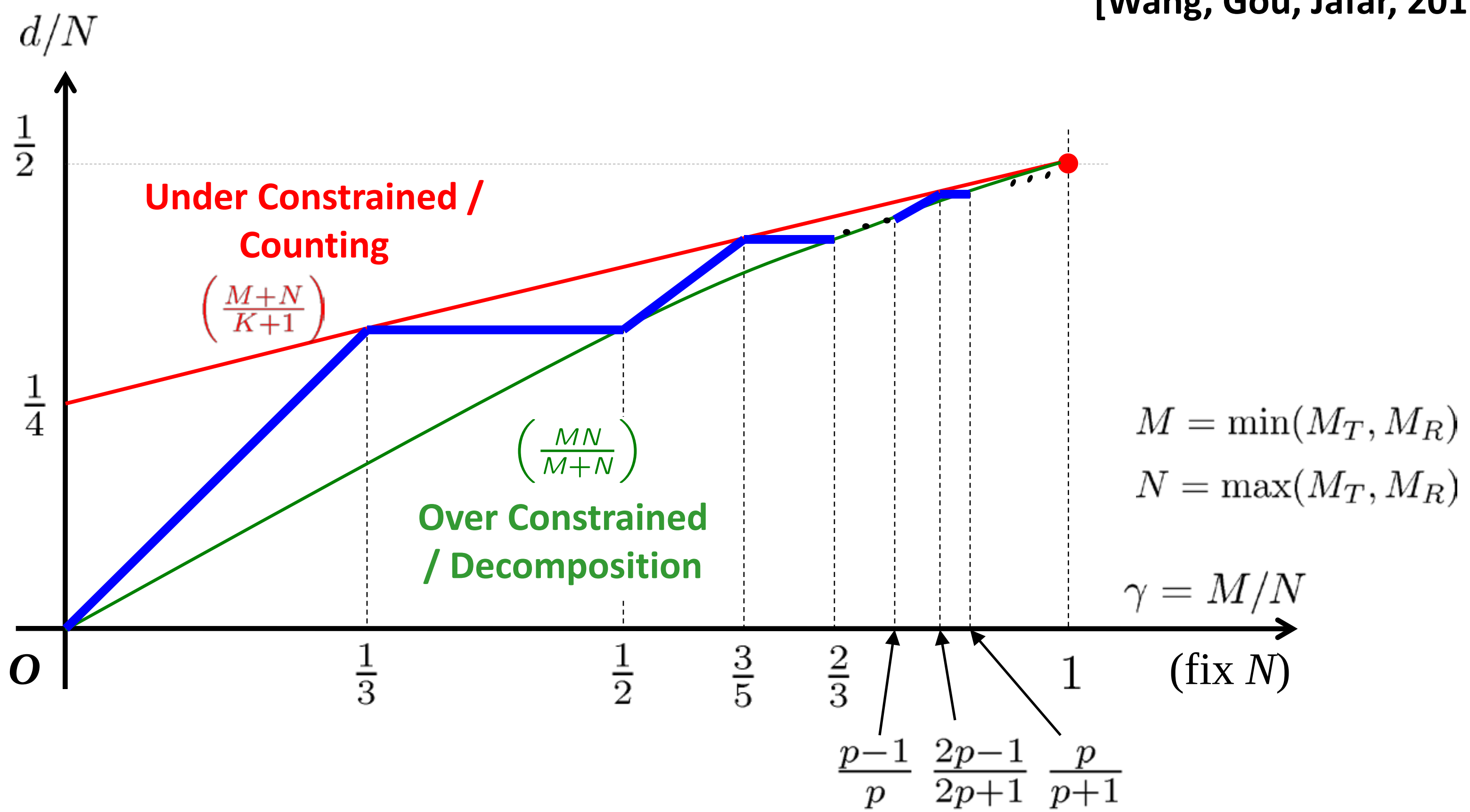
N antennas at each receiver



Linear Dimension Counting Arguments

DoF of the K=3 User MIMO IC

[Wang, Gou, Jafar, 2011]

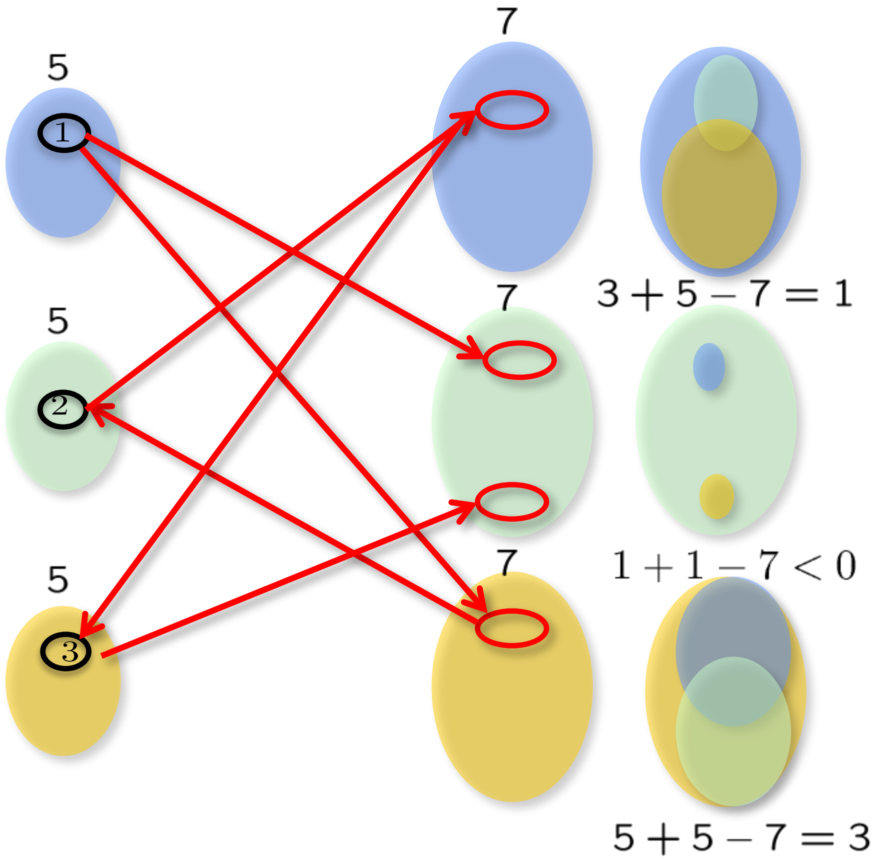


The DoF value is **piecewise linear** depending on *M* and *N*, alternatively.

(1) The counting outer bound is achievable when $M/N = \{1/3, 3/5, 5/7, 7/9, \cdots\}$

(2) The decomposition inner bound is optimal when $M/N = \{1/2, 2/3, 3/4, 4/5, \cdots\}$

Subspace Alignment Chains



$M_T = 5$ $M_R = 7$

$\kappa = 3$

$$d = \min\left(\frac{M}{2 - 1/\kappa}, \frac{N}{2 + 1/\kappa}\right)$$

$$N = \max(M_T, M_R)$$

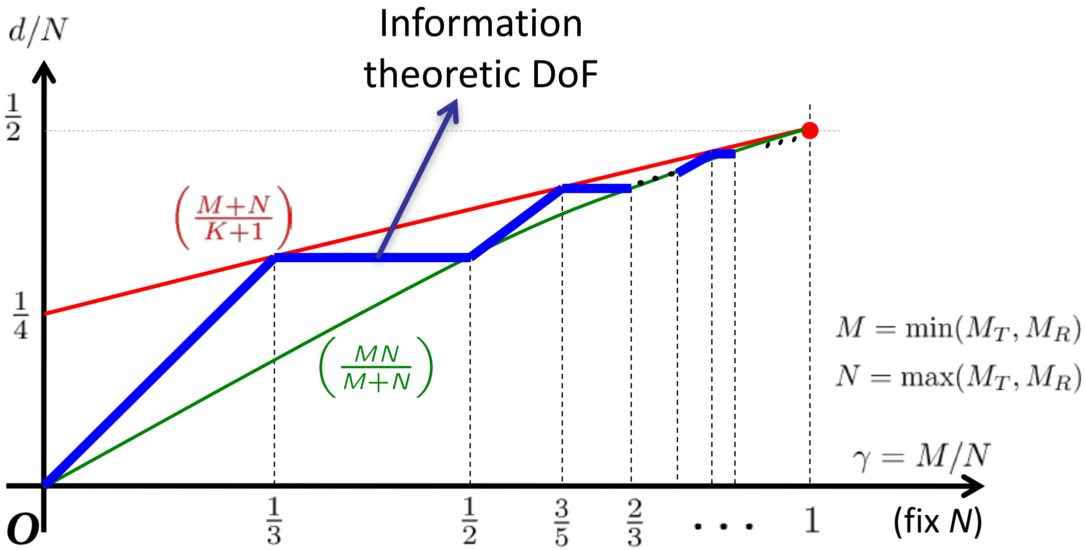
$$M = \min(M_T, M_R)$$

$$\kappa = \lceil \tfrac{M}{N-M} \rceil$$

Length of the alignment chain

Subspace Alignment Chain

Observations of the DoF Results (K=3)



(1) Antenna Redundancies

$$d = \min\left(\frac{M}{2-1/\kappa}, \frac{N}{2+1/\kappa}\right)$$

Only the N-bound/M-bound is tight **=>** transmitter/receiver antenna redundancies.

e.g., both (M, N) = (9, 15) and (M, N) = (10, 15) have 6 DoF per user.

(2) DoF Benefit of MIMO Processing

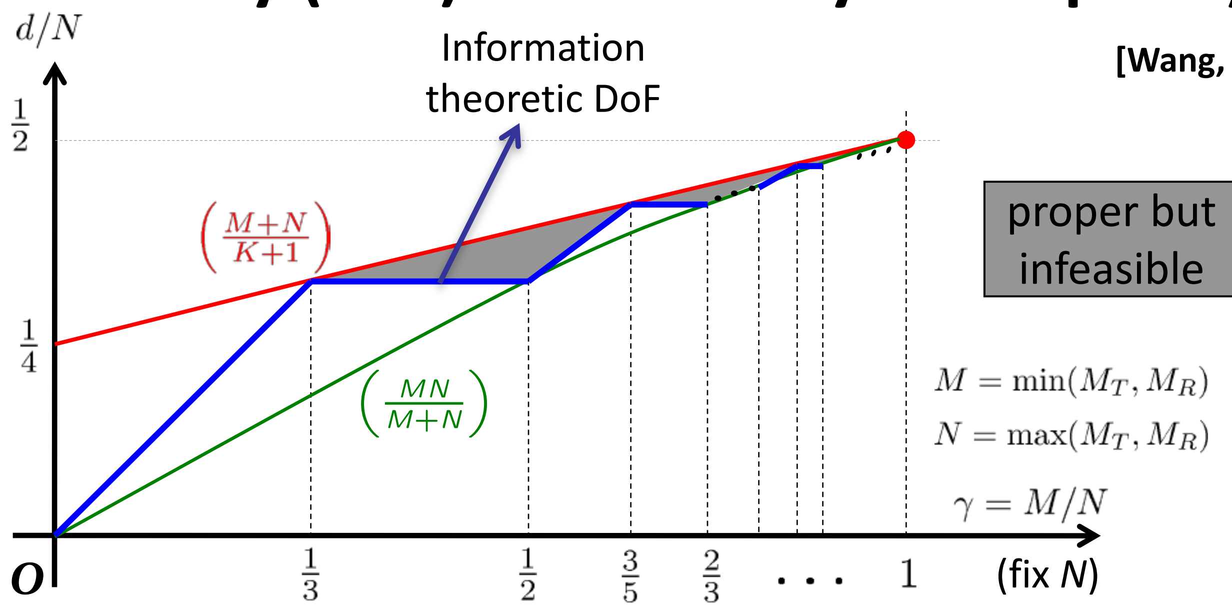
$$d > \frac{MN}{M+N}$$ (green curve)

By decomposing MIMO to SISO, no cooperation among the antennas at each node.

Except for $M/N = \{1/2, 2/3, 3/4, 4/5, \cdots\}$

Linear Feasibility (K=3): Infeasibility of Proper Systems

[Wang, Gou, Jafar, 2011]



Improper => Infeasible

[Bresler, Cartwright, Tse, 2011]

[Razaviyayn, Lyubeznik, Luo, 2011]

Proper => Feasible ?

(1) proper = feasible, if *M*, *N* are divisible by *d*.

[Razaviyayn, Lyubeznik, Luo, 2011]

(2) proper = feasible, if *M* = *N*.

[Bresler, Cartwright, Tse, 2011]

We fully solve the feasibility problem for the K = 3 setting!

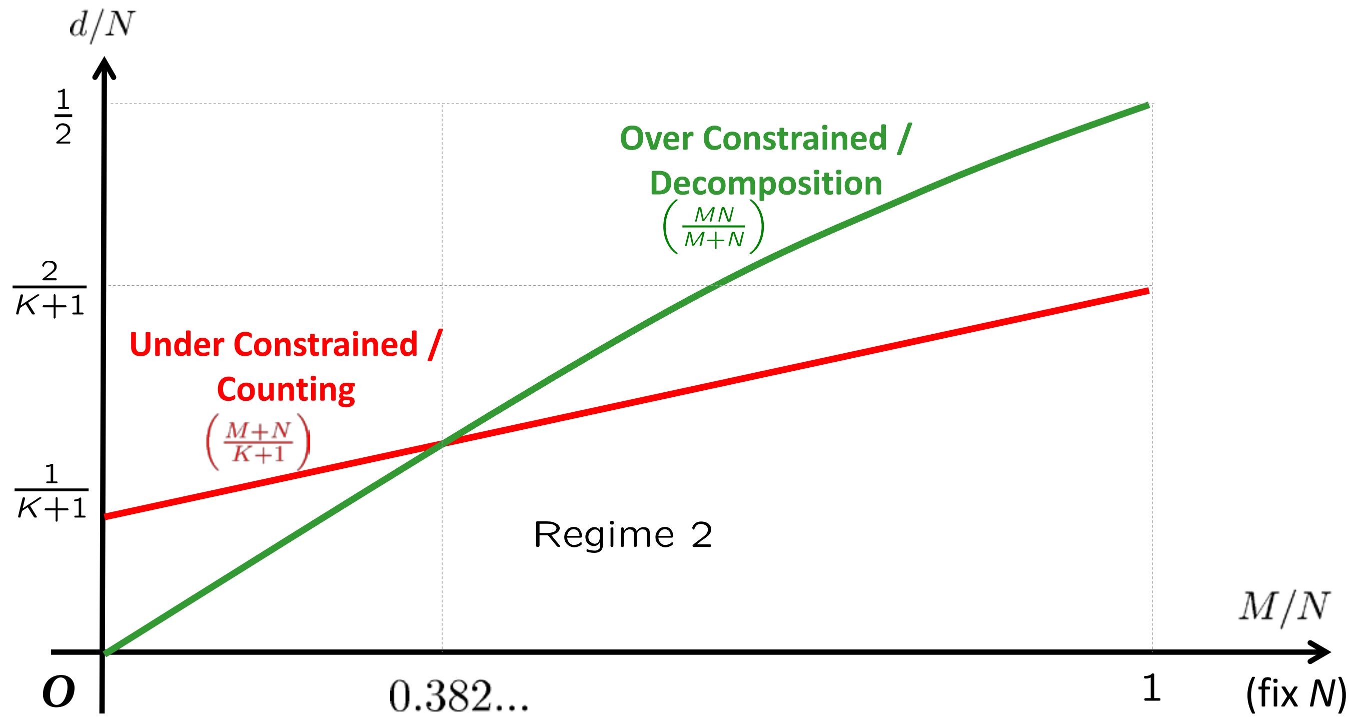
(3) proper = feasible, if $M/N = \{3/5, 5/7, 7/9, \cdots\}$

$$d^* = \left\lfloor \min\left(\frac{M}{2-1/\kappa}, \frac{N}{2+1/\kappa}\right) \right\rfloor$$

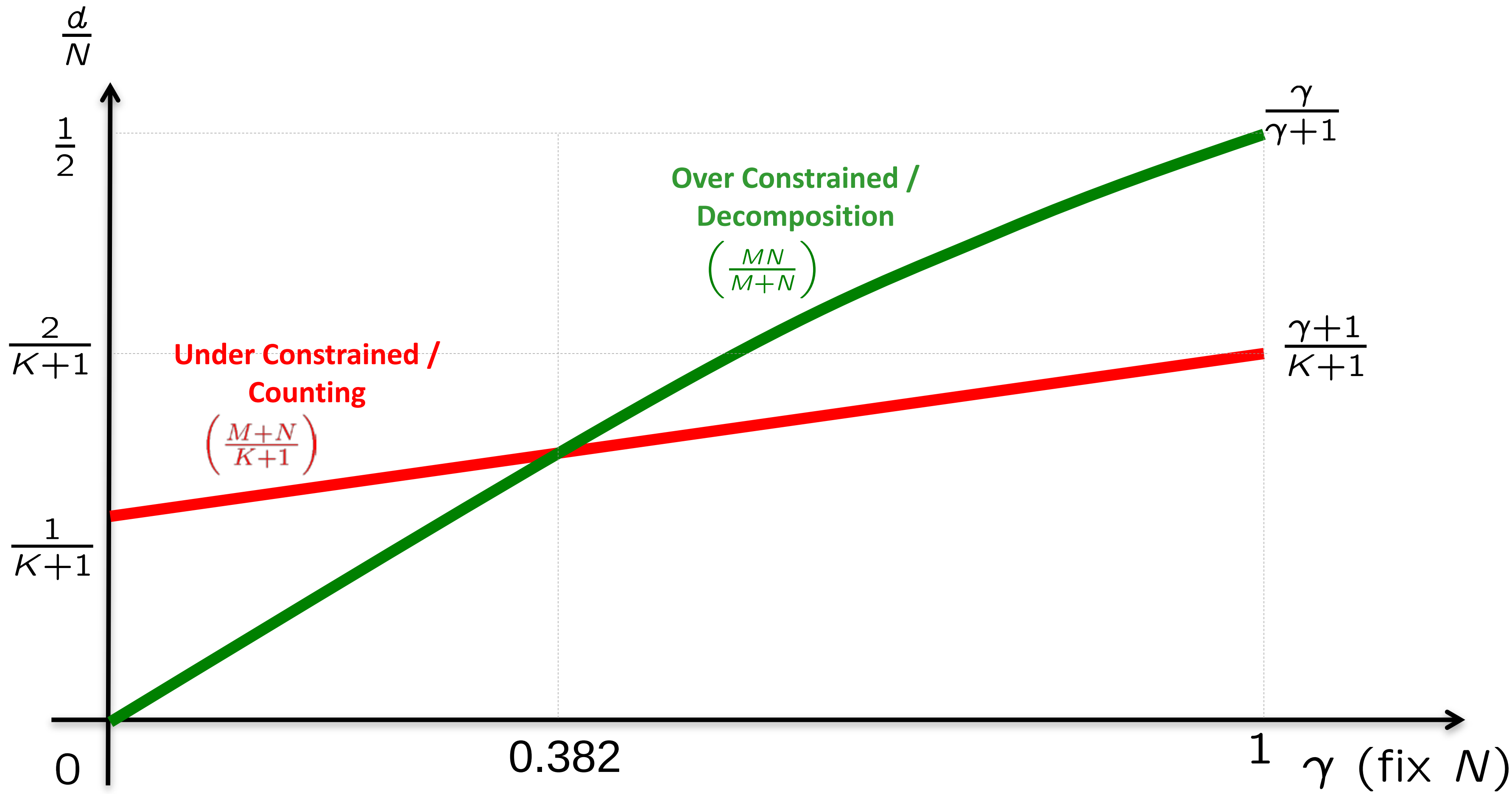
Many proper systems and most strictly proper systems are infeasible.

Understanding Regime 2: Information Theoretic DoF

K > 3 User MIMO Interference Channel



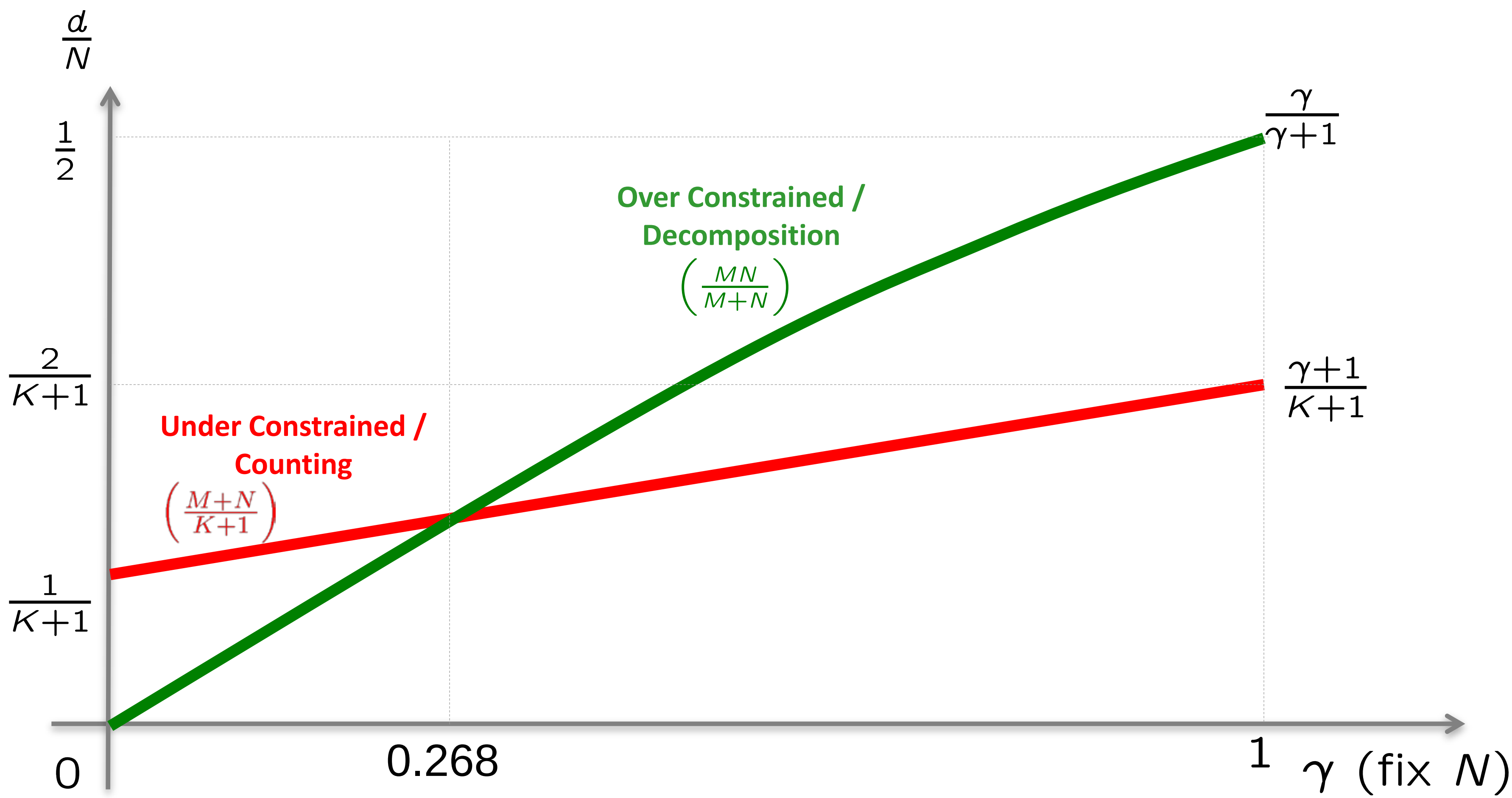
K = 4 User MIMO IC



$$\gamma = \frac{M}{N}$$

$(M \leq N)$

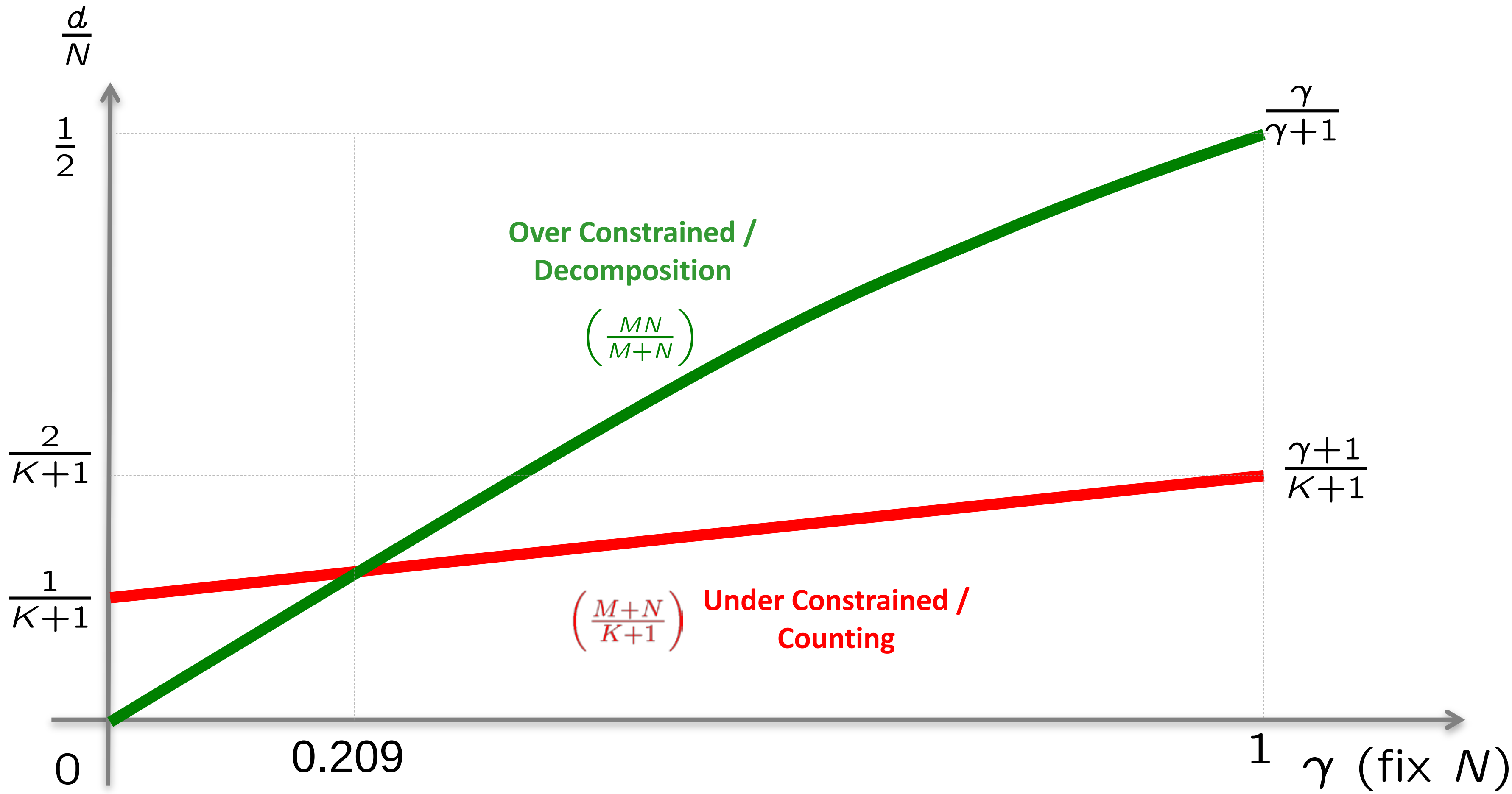
K = 5 User MIMO IC



$$\gamma = \frac{M}{N}$$
$$(M \leq N)$$

上午1时1分

K = 6 User MIMO IC



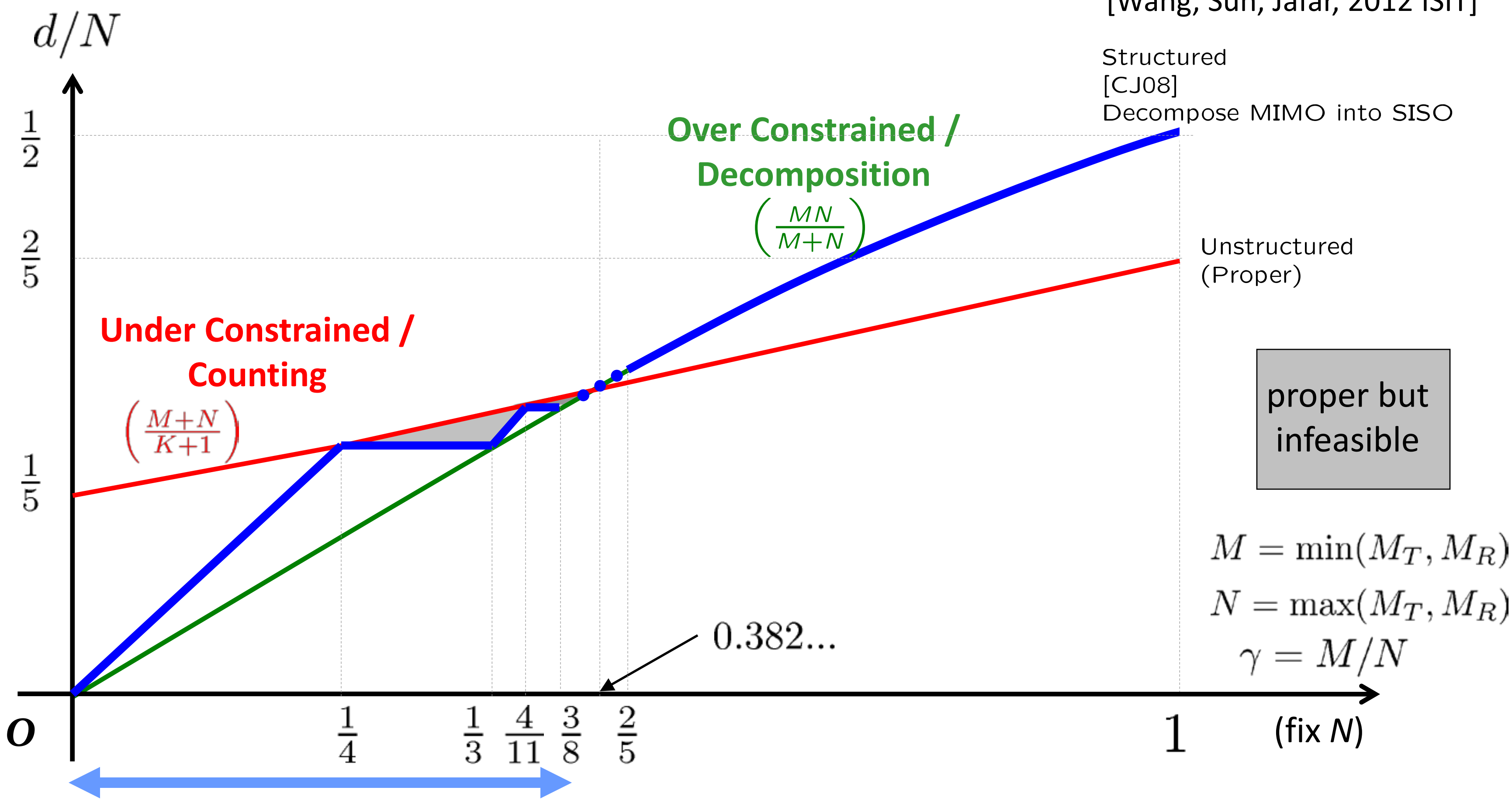
$$\gamma = \frac{M}{N}$$

$$(M \leq N)$$

上午1时1分

DoF of the K = 4 User MIMO IC

[Wang, Sun, Jafar, 2012 ISIT]



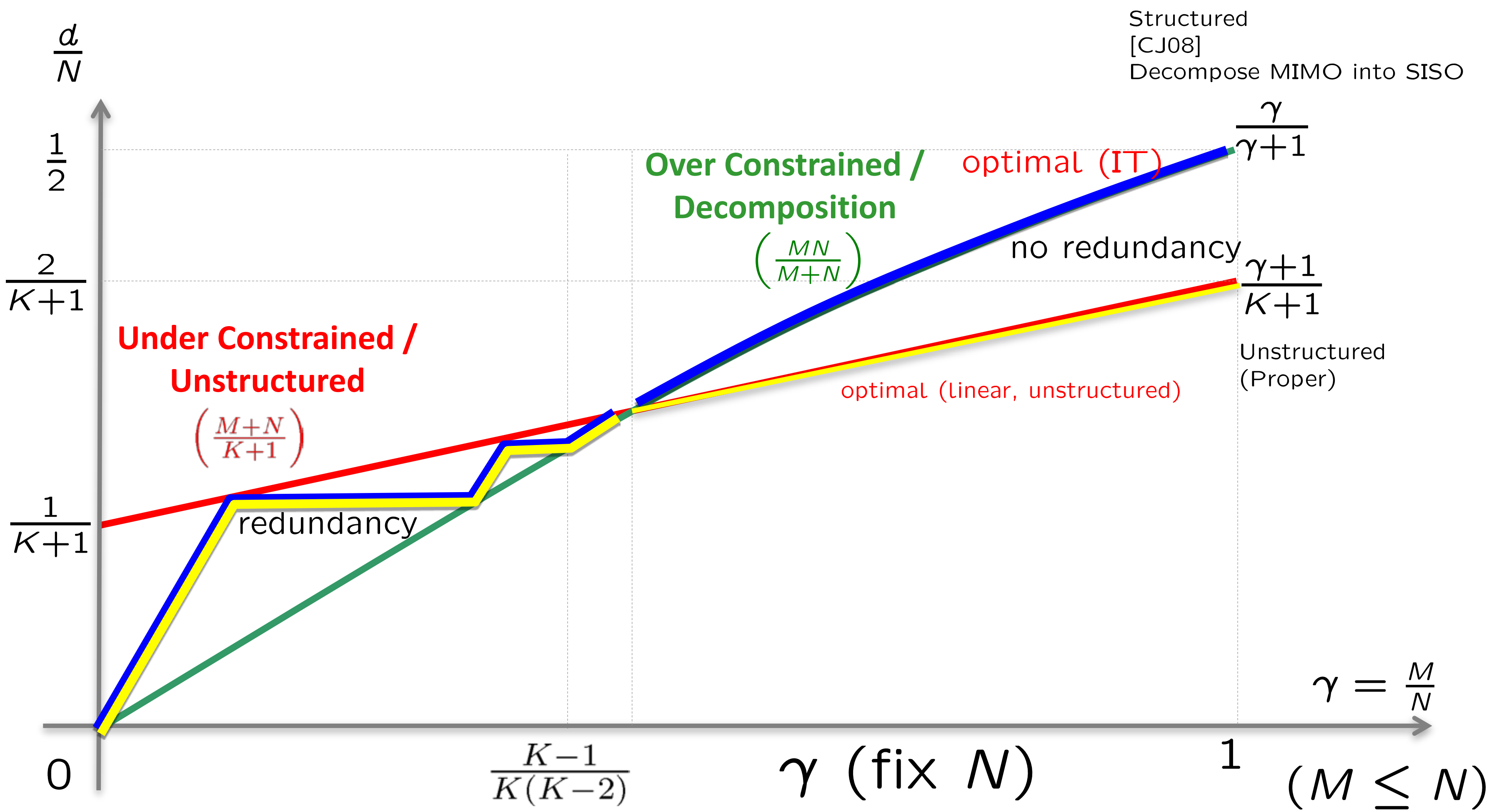
We need both DoF achievability and DoF Converse.

Subspace Alignment Chains

We only need DoF **Converse** !

Genie Chains

K User $M_T \times M_R$ MIMO Interference Channel

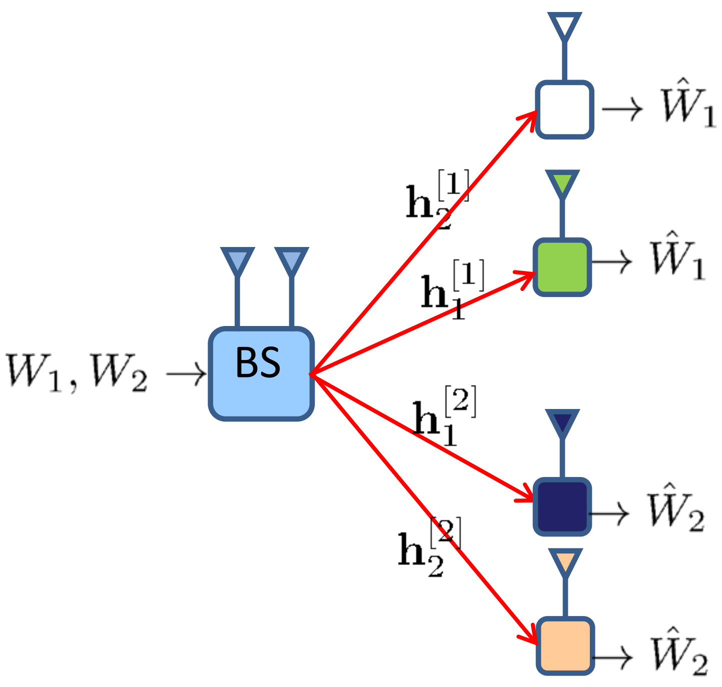


If the channel has finite number of states,

the transmitter knows the collection of finite channel states,

but it does not know its exact state,

Channel Uncertainty

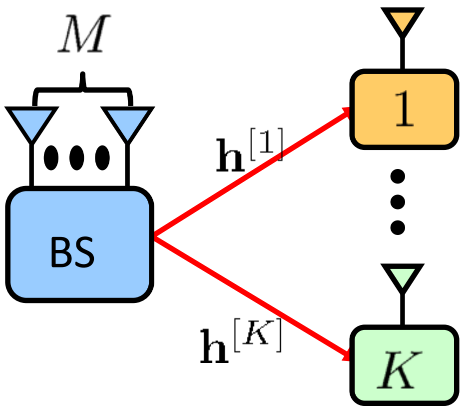


DoF = 2

DoF = 4/3 < 2

Compound MISO Broadcast Channel

M antennas at BS, K users, J generic channel states per user



$$\mathbf{h}^{[k]} \in \left\{\mathbf{h}_1^{[k]}, \cdots, \mathbf{h}_J^{[k]}\right\}$$

- Conjecture: **[Weingarten, Shamai, Kramer, ITA 07]**

  DoF$\to$ 1 as J increases

- Settled the conjecture in negative:

  [Gou, Jafar, Wang, 2011 TIT]

$$\boxed{\mathrm{DoF} = \frac{MK}{M+K-1}}$$

Optimal !

The value does not depend on the number of channel states!!

Challenge: arbitrary large number of alignment constraints

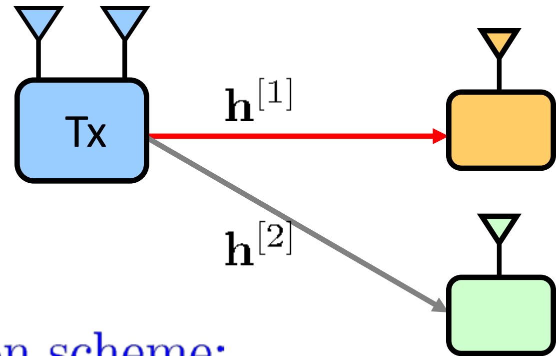
Overcome: asymptotic alignment for K user SISO Interference Channel,
IA in rational dimensions/time-varying

New insights: **aligning instead of avoiding interference**

Blind Interference Alignment (BIA)

If we have No CSIT?

[Jafar 09]

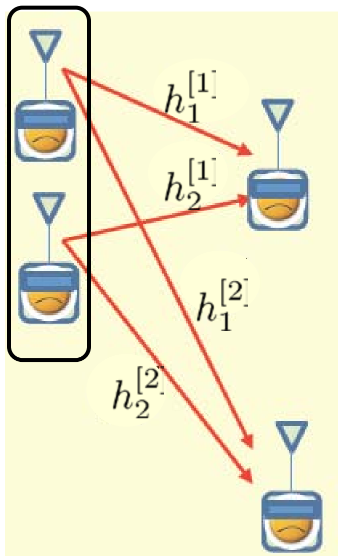


Desired User: $\mathbf{h}^{[1]}(1)$ | $\mathbf{h}^{[1]}(2)$

Undesired User: $\mathbf{h}^{[2]}(1)$ | $\mathbf{h}^{[2]}(1)$

Transmission scheme:

$t=2$	$t=1$
x_1	x_1
x_2	x_2



$t=1$	$t=2$
$h_1^{[1]}(1)x_1 + h_2^{[1]}(1)x_2$	$h_1^{[1]}(2)x_1 + h_2^{[1]}(2)x_2$
$h_1^{[2]}(1)x_1 + h_2^{[2]}(1)x_2$	$h_1^{[2]}(1)x_1 + h_2^{[2]}(1)x_2$

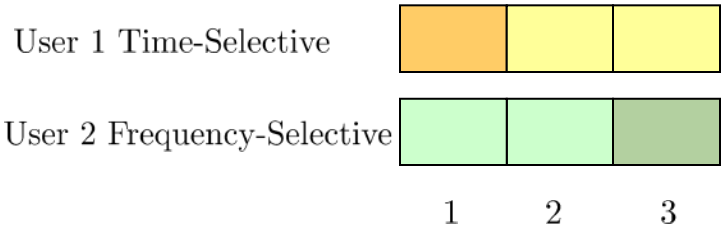
Idea: **Repeat symbols where desired users' channel changes,**

while undesired users' channels remain the same.

Channel Coherence Structure Enables BIA

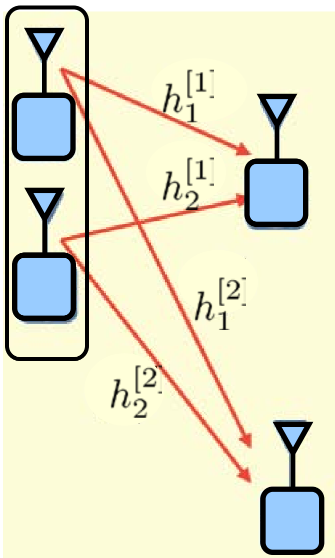
[Jafar 09]

- No CSIT: DoF $= \frac{4}{3}$



The super-symbol structure used for BIA arises naturally in certain practical settings.

$t=3$	$t=2$	$t=1$
$x_1^{[2]}$	$x_1^{[1]}+x_1^{[2]}$	$x_1^{[1]}$
$x_2^{[2]}$	$x_2^{[1]}+x_2^{[2]}$	$x_2^{[1]}$

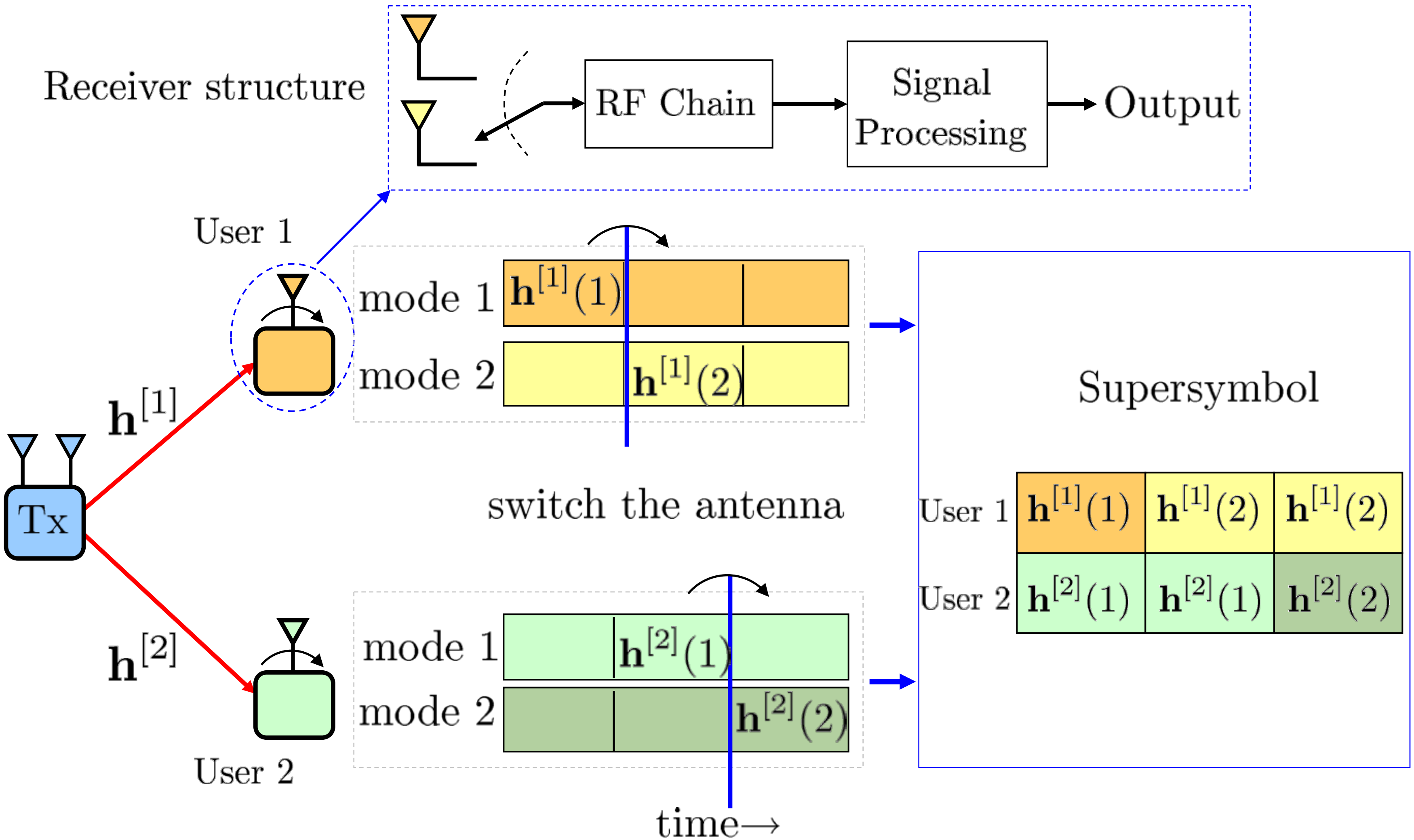


$t=1$	$t=2$	$t=3$
$h_1^{[1]}(1)x_1^{[1]} + h_2^{[1]}(1)x_2^{[1]}$	$h_1^{[1]}(2)x_1^{[1]} + h_1^{[1]}(2)x_1^{[2]} + h_2^{[1]}(2)x_2^{[1]} + h_2^{[1]}(2)x_2^{[2]}$	$h_1^{[1]}(2)x_1^{[2]} + h_2^{[1]}(2)x_2^{[2]}$
$h_1^{[2]}(1)x_1^{[1]} + h_2^{[2]}(1)x_2^{[1]}$	$h_1^{[2]}(1)x_1^{[1]} + h_1^{[2]}(1)x_1^{[2]} + h_2^{[2]}(1)x_2^{[1]} + h_2^{[2]}(1)x_2^{[2]}$	$h_1^{[2]}(2)x_1^{[2]} + h_2^{[2]}(2)x_2^{[2]}$

If the channel coherence does not display any special structure, can we create the opportunities to enable BIA?

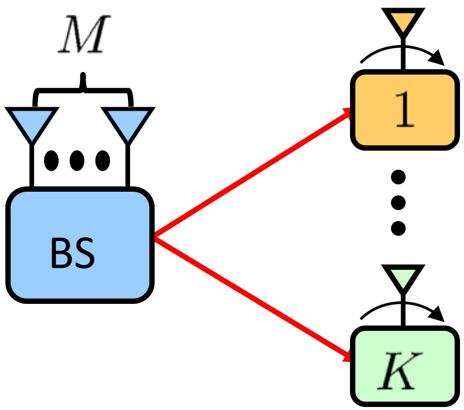
Staggered Antenna Switching

[Gou, Wang, Jafar, 2011 TSP]



K User MISO BC with No CSIT

[Gou, Wang, Jafar, 2011 TSP]



$$\text{DoF} = \frac{MK}{M+K-1}$$

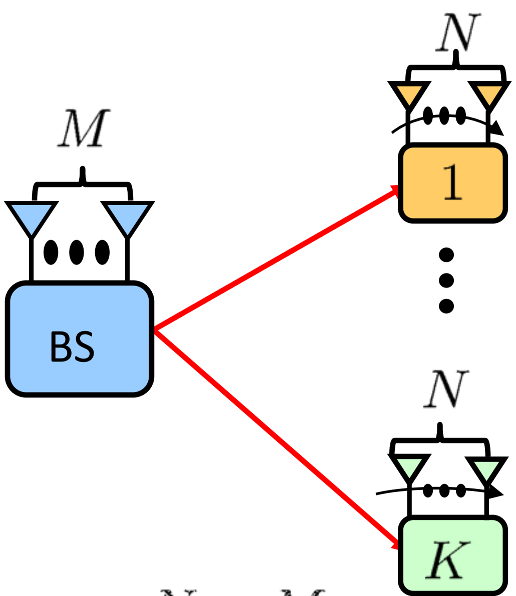
Optimal !

- Achieve the outer bound
- DoF$\rightarrow M$ as $K \rightarrow \infty$
- Same as full CSIT: $\min(M, K) = M$ as $K \rightarrow \infty$
- No CSIR is needed for antenna switching and nulling interference.

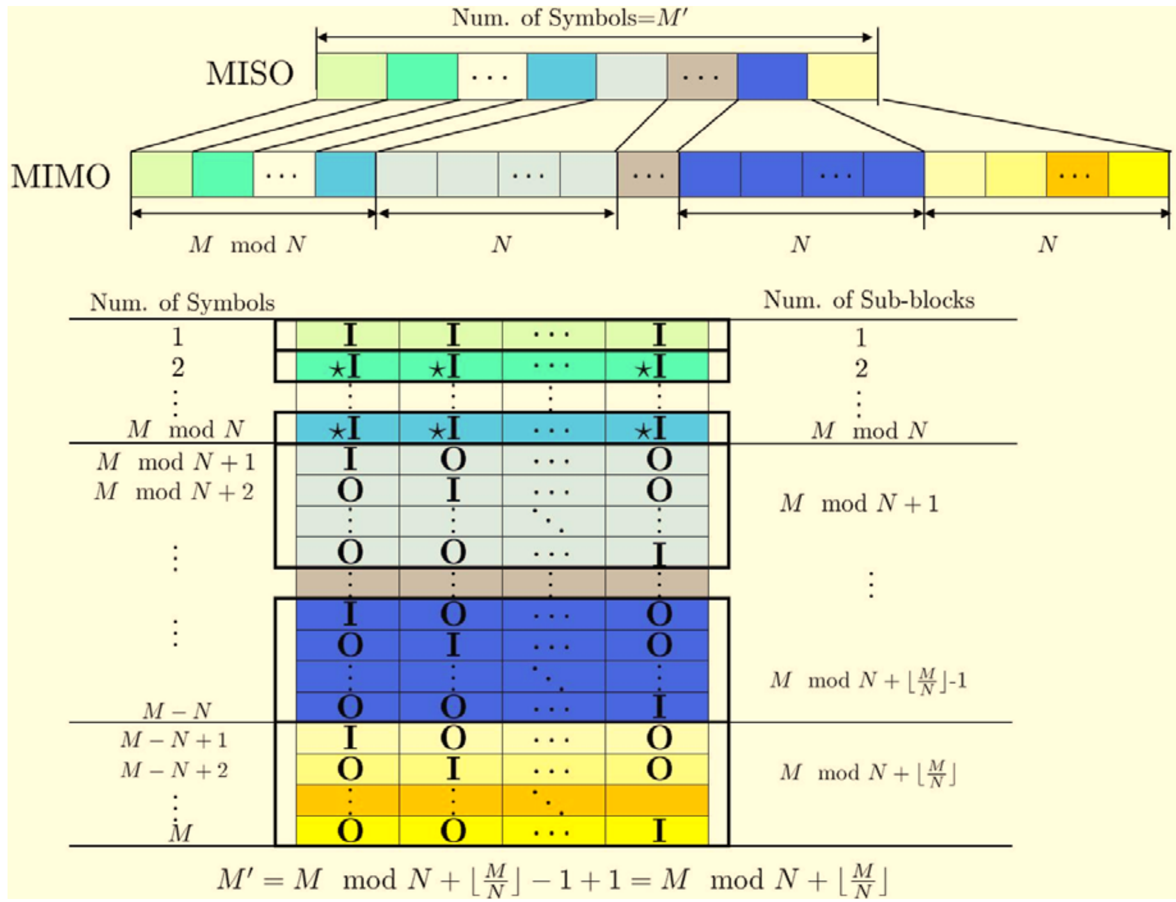
K User MIMO BC with No CSIT

[Wang, Gou, Jafar, 2010 Asilomar]

$$\text{DoF} = \frac{MNK}{M + KN - N}$$

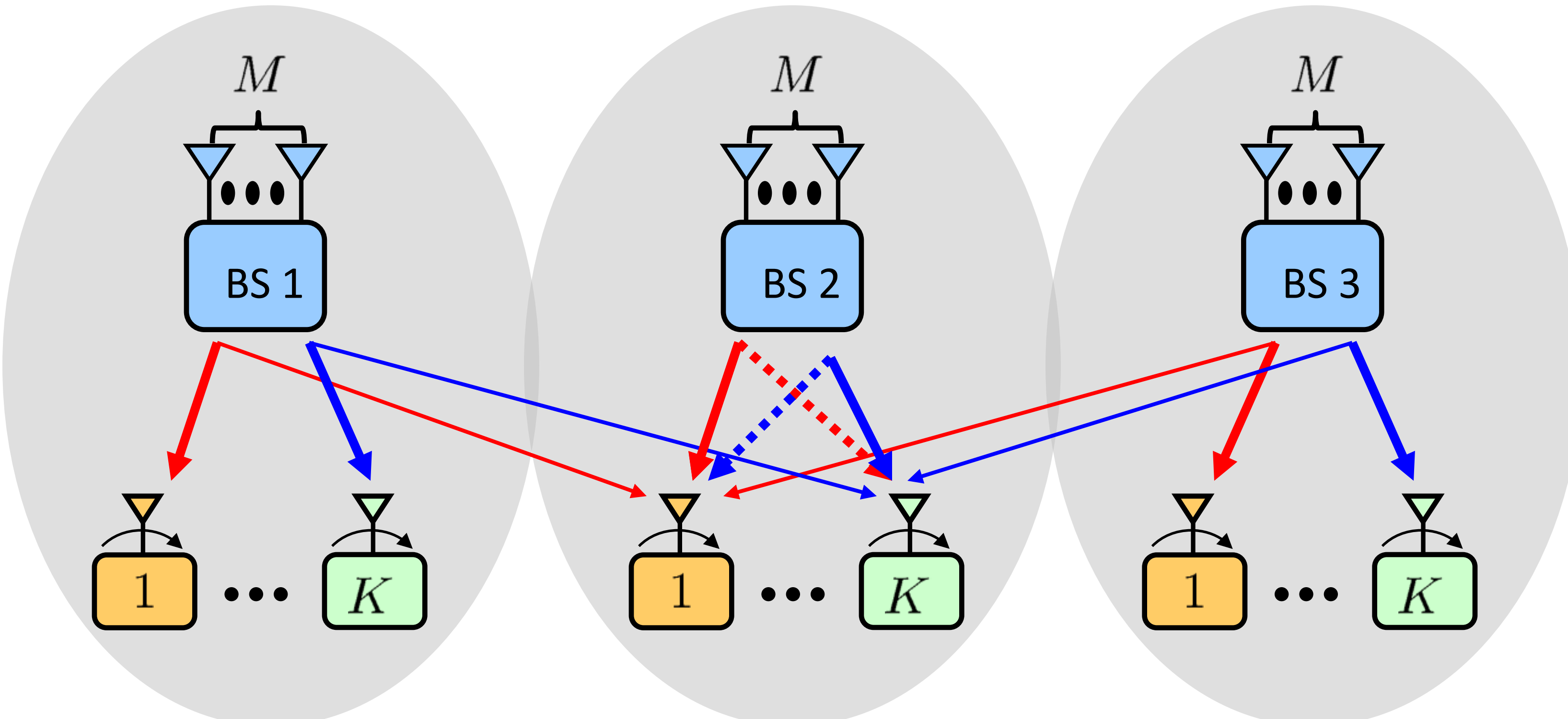


$N < M$



Applications: BIA in the Cellular Networks

[Wang, Papadopoulos, Ramprashad, Caire, 2011 ITA, ICC]



In some ways the operating principles of BIA resemble that of CDMA codes

- In CDMA *bandwidth* is used to generate user specific code. (to separate streams)

- In BIA *antenna modes* are used at each receiver to enable "user-specific" codes. (to align intra- and inter-cell interference)

Closing Remark

Network Interference Management via the Interference Alignment approach.

We introduce the degrees of freedom (DoF) results of:

[Interference Alignment with Asymmetric Complex Signaling]

[K User MIMO Gaussian Interference Channel]

[Compound Broadcast Channel]

[Blind Interference Alignment and Applications]

We expect that the new insights behind all the results contribute to the design of future communications systems.